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Note on Hansen's General Formulae for Perturbations.

BY G. W. HILL.

The last form in which HANSEN expressed the perturbations of the mean anomaly and equated radius vector is exhibited by the following equations:

$$n_0 z = n_0 t + c_0 + \int \left\{ \overline{W} + \frac{h_0}{h} \left(\frac{\nu}{1+\nu} \right)^2 \right\} n_0 dt,$$

$$\nu = C - \frac{1}{2} \int \left(\frac{d\overline{W}}{d\zeta} \right) dt,$$

(Equations 36 and 37, p. 97.)[*]

It will be perceived that the right-hand member of the first of these involves three quantities, viz. $\overline{W}$, ν and $\frac{h_0}{h}$. But the last of these quantities has no share in defining the position of the body, and it is desirable to get rid of it, provided that can be done without complicating the equation. This is readily accomplished by means of the equation (33, p. 95)

$$\frac{dz}{dt} = \frac{h_0}{h(1+\nu)^2}.$$

The result is

$$n_0 z = n_0 t + c_0 + \int \frac{\overline{W} + \nu^2}{1 - \nu^2} n_0 dt.$$

Why HANSEN has not put the equation in this form I cannot imagine; the advantage, not only as regards simplicity of expression, but also in point of ease of computation, is evident.

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[*] See *Auseinandersetzung einer zweckmässigen Methode zur Berechnung der absoluten Störungen der kleinen Planeten. Von P. A. Hansen. Erste Abhandlung.* (*Abhandlungen der Königlichen Sächsischen Gesellschaft der Wissenschaften. Band* III.) The numbering of the equations and the paging are from this volume.

HANSEN develops $\overline{W}$ by TAYLOR'S theorem, and, limiting ourselves to the second power of the disturbing force, we have

$$\overline{W} = \overline{W}_0 + \left(\frac{d\overline{W}_0}{d\gamma}\right) n_0 \delta z = \overline{W}_0 - 2\frac{d\nu}{dt}\delta z\,.$$

When this value is substituted for $\overline{W}$ in the equation for $n_0 z$, we have a differential equation of the first order and degree for the determination of δz, the integral of which is well known. Terms of three dimensions with respect to disturbing forces being neglected, this procedure furnishes the equation

$$n_0 \delta z = (1 - 2\nu)\int [(1 + 2\nu)\,\overline{W}_0 + \nu^2]\, n_0 dt\,,$$

which, however, is without interest other than analytical, as its use involves more labor than that of the equation given by HANSEN.

HANSEN'S equation for the determination of ν has the disadvantage of not affording the constant term of this quantity, and is inconvenient in computing the portion, of the form

$$At + Bt^2 + Ct^3 + \ldots\,,$$

which is independent of the arguments g, g', &c., as the values of A, B, &c., must be determined to a degree of accuracy much beyond what is necessary in the case of the other terms. As all the arbitrary constants admissible have been introduced by the integrations which give z, it is evident there must exist an equation determining ν without additional integrations. HANSEN has virtually employed this in the place where he shows how the constant term of ν is to be obtained, but has nowhere given it explicitly. This lacuna I propose to fill here.

The equation 39, p. 97,

$$W_0 = 2\frac{h}{h_0} - \frac{h_0}{h} - 1 + 2\frac{h}{h_0}\xi\frac{\rho}{a_0}\cos\omega + 2\frac{h}{h_0}\eta\frac{\rho}{a_0}\sin\omega\,,$$

may be employed to discover the value of $\frac{h}{h_0}$. The known expressions for $\frac{\rho}{a_0}\cos\omega$ and $\frac{\rho}{a_0}\sin\omega$ are

$$\frac{\rho}{a_0}\cos\omega = -\frac{3}{2}e + \left(J^{(0)}_{\frac{e}{2}} - J^{(2)}_{\frac{e}{2}}\right)\cos\gamma + \frac{1}{2}\left(J^{(1)}_{e} - J^{(3)}_{e}\right)\cos 2\gamma + \ldots\,,$$

$$\frac{\rho}{a_0}\sin\omega = \left(J^{(0)}_{\frac{e}{2}} + J^{(2)}_{\frac{e}{2}}\right)\sin\gamma + \frac{1}{2}\left(J^{(1)}_{e} + J^{(3)}_{e}\right)\sin 2\gamma + \ldots\,,$$

where HANSEN'S notation for the BESSELIAN function is employed, and the subscript zero, which properly belongs to e, is, for convenience in writing, omitted. In his memoirs, where the mean anomaly is employed as the independent variable, HANSEN directs to compute only the parts of W_0 which are independent of γ or which have $\pm\,\gamma$ in their arguments; that is, the parts which have the form

$$X_0 + X_1 \cos \gamma + X_2 \sin \gamma\,,$$

X_0, X_1 and X_2 being independent of γ.

It will be easily perceived that, if we put

$$P = \frac{3}{2}\,\frac{e}{J^{(0)}_{\frac{e}{2}} - J^{(2)}_{\frac{e}{2}}}\,,$$

P being thus a constant, the three first terms of W_0 must have the value

$$2\frac{h}{h_0} - \frac{h_0}{h} - 1 = X_0 + PX_1.$$

In this equation we may substitute for $\frac{h_0}{h}$ its value obtained from equation 33, and thus we obtain

$$\frac{2}{(1+\nu)^2\left(1+\frac{d\,.\,\delta z}{dt}\right)} - (1+\nu)^2\left(1+\frac{d\,.\,\delta z}{dt}\right) - 1 = X_0 + PX_1.$$

This equation, when $\frac{d\,.\,\delta z}{dt}$ is known, gives ν without additional integrations. To put it into a form suitable for computation, we add to each member such a quantity as will make the first equal to $-\,6\nu$, then dividing both members by $-\,6$ we get

$$\nu = -\frac{1}{6}\left[X_0 + PX_1\right] - \frac{1}{2}\left[(1+\nu)^2\frac{d\,.\,\delta z}{dt} + \nu^2\right] + \frac{\left[(1+\nu)^2\frac{d\,.\,\delta z}{dt} + 2\nu + \nu^2\right]^2}{3\,(1+\nu)^2\left(1+\frac{d\,.\,\delta z}{dt}\right)}\,.$$

This equation is rigorous. If we may restrict ourselves to terms of the first order with respect to disturbing forces, it reduces to

$$\nu = -\frac{1}{6}\left[X_0 + PX_1\right] - \frac{1}{2}\,\frac{d\,.\,\delta z}{dt}\,,$$

or, if terms of the second order must be included, to

$$\nu = -\frac{1}{6}\left[X_0 + PX_1\right] - \frac{1}{2}\frac{d\,.\,\delta z}{dt} + \frac{1}{3}\left[\frac{d\,.\,\delta z}{dt} + \frac{1}{2}\nu\right]^2 + \frac{3}{4}\nu^2\,.$$

The function usually tabulated is com. log $(1 + \nu)$; and we have com. log $(1 + \nu) = M\left\{-\frac{1}{6}\left[X_0 + PX_1\right] - \frac{1}{2}\frac{d\,.\,\delta z}{dt} + \frac{1}{3}\left[\frac{d\,.\,\delta z}{dt} + \frac{1}{2}\nu\right]^2 + \frac{1}{4}\nu^2\right\}$, M being the modulus of common logarithms.

These equations are as readily used as those given by HANSEN, and are free from the disadvantages, previously mentioned, which belong to the latter. All the quantities involved, except X_1, have already been obtained in the computation of δz. Also X_1 is readily got by putting $\gamma = 0$ in the terms of W_0 which involve this quantity, and summing two and two together the terms which result.

WASHINGTON, D. C., *December* 17, 1881.